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The measurement of turbulence intensities using real-time† laser-Doppler velocimetry

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A technique by which the effect of the random phase fluctuations or Doppler ambiguity can be removed from measurements of turbulence intensities by the laser-Doppler velocimeter is proposed. Because the ambiguity is often much broader band than the turbulence, it is proposed that a sequence of low-pass filters at frequencies above those of the turbulence be used in the measurement of turbulence intensities. The true turbulence intensity is given by the extrapolation of these measurements to zero frequency.

The measurement of unsteady fluid velocities with a laser-Doppler velocimeter (LDV) is difficult because of the presence of the Doppler ambiguity. In essence, the Doppler ambiguity arises from the random arrival and departure of the scattering particles in the sampling volume. Its simplest manifestation is the finite width of the power spectrum of the Doppler beat current in a steady, uniform, laminar flow (George & Berman 1973). Turbulence, velocity gradients, refractive-index fluctuations and noise also contribute to this broadening. George & Lumley (1973) have discussed the effect of this broadening on the Doppler signal and have presented an exact theory for the Doppler ambiguity for both the Doppler beat signal and its demodulation.

The problem when measuring turbulence intensities is particularly acute. In spite of the fact that the outer envelope of the Doppler current spectrum is determined in part by the probability density of the turbulence, accurate correction is difficult (if not impossible) because the turbulence contributes to both the fluctuations of the centre-frequency (the quantity sought) and the ambiguous broadening (cf. George & Lumley, p. 338). This is illustrated in figure 1 (*a*).

At first glance, the demodulated signal presents even greater problems. The mean-square demodulated signal may be written as

$$\text{mean-square output} = K^2\overline{u^2}+\overline{\phi^2}, \tag{1}$$

where $\overline{u^2}$ is the mean-square fluctuating velocity, $\overline{\phi^2}$ is the mean-square phase fluctuation due to ambiguity, and K is the scattering wavenumber, which is

† The term real-time has been adopted to distinguish this from discrete-realization velocimetry.

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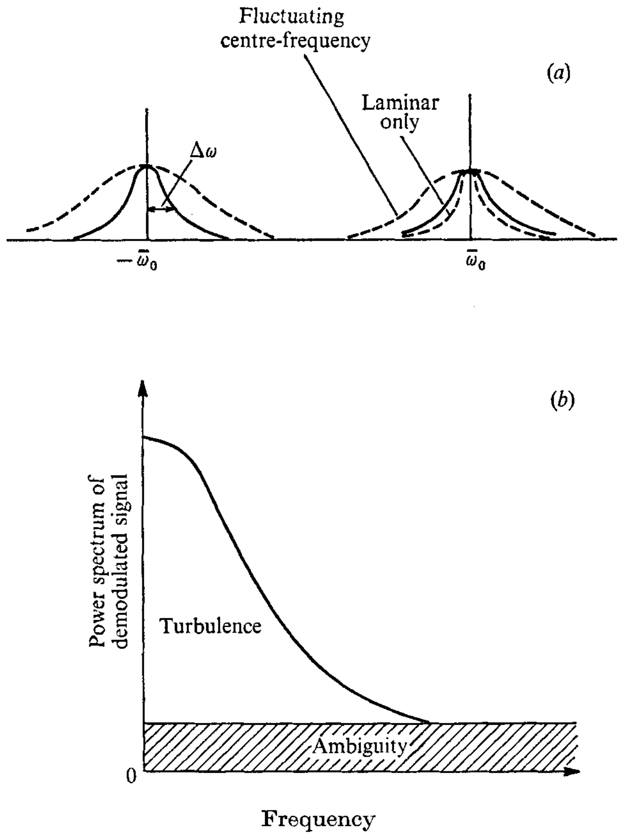

FIGURE 1. (*a*) Power spectrum of the Doppler beat current, showing contributions of turbulence and ambiguity. (*b*) Power spectrum of demodulated Doppler signal, showing contributions from turbulence and ambiguity. Ambiguity eventually rolls off as one over frequency.

simply the mean Doppler frequency divided by the mean velocity. The mean-square fluctuations due to ambiguity are in principle infinite; hence, attempts to measure turbulence intensities are catastrophic. The mean-square ambiguity fluctuation is rendered finite by the low-pass response of the instrumentation; nevertheless, the ambuiguity contribution to the overall intensity measured may be significant (if not dominant) since the spectra of the turbulence and the ambiguity share the same frequency band. The spectra of the demodulated turbulent signal and the ambiguity phase fluctuations are illustrated graphically in figure 1 (*b*).

This paper represents a slight extension of the work of George & Lumley (1973) and proposes a technique by which accurate turbulence intensity measurements can be made using the demodulated signal. It will be explicitly assumed that the effective scattering volume is sufficiently small that intensity losses due to spatial averaging across the volume are not a problem (cf. George & Lumley 1973, p. 329).

The spectrum of the demodulated Doppler ambiguity for a Gaussian sample volume is

$$N(\omega) = \frac{\Delta\omega}{4\pi^{\frac{1}{2}}} \sum_{n=1}^{\infty} n^{-\frac{3}{2}} \exp\left[\frac{-\omega^2}{4n(\Delta\omega)^2}\right], \tag{2}$$

where $\Delta\omega$ is the total Doppler-ambiguity bandwidth (see George & Lumley 1973, p. 343; also George & Berman 1973; Berman & Dunning 1973) and the dimensions of $N(\omega)$ are $(\mathrm{rad/s})^2/(\mathrm{rad/s})$. In (2), n is simply a summing index and results from a Fourier-transformed logarithm (see George & Lumley 1973). Note that, because of the linearity of the laser-Doppler velocimeter, $N(\omega)$ is readily converted to the appropriate velocity dimensions by multiplication by the mean velocity squared divided by the mean Doppler beat frequency squared. It is easy to show that the integral of $N(\omega)$ over all frequencies is infinite, rendering intensity measurement impossible.

The contribution of the ambiguity phase fluctuations can be made as small as we like by passing the demodulated signal through a low-pass filter; however at some point we shall begin also to filter out the turbulence fluctuations. Clearly the relative contribution is at a minimum when the filter frequency is near the frequency at which the turbulence spectrum and ambiguity spectrum are equal. It will be shown below that, by using filters at several frequencies, the true turbulence intensity can be obtained by extrapolation to zero filter frequency.

The system response function for a simple low-pass (R.C.) filter is given by (Jenkins & Watts 1968)

$$|H(\omega)|^2 = 1/[1+(\omega/\omega_L)^2], \tag{3}$$

where ω_L is the cut-off frequency and is given by $\omega_L = 1/RC$. The low-passed ambiguity can then be written as

$$N_{LP}(\omega) = \frac{\Delta\omega}{4\pi^{\frac{1}{2}}} \sum_{n=1}^{\infty} n^{-\frac{3}{2}} \frac{\exp[-\omega^2/4n(\Delta\omega)^2]}{1+(\omega/\Delta\omega)^2(\Delta\omega/\omega_L)^2}. \tag{4}$$

For convenience, we shall non-dimensionalize $N(\omega)$, ω and ω_L by $\Delta\omega$ as follows:

$$\alpha = \omega/\Delta\omega, \quad \alpha_L = \omega_L/\Delta\omega, \quad \tilde{N}(\alpha) = N(\omega)/\Delta\omega. \tag{5}$$

Equation (4) becomes

$$\tilde{N}_{LP}(\alpha) = \frac{1}{4\pi^{\frac{1}{2}}} \sum_{n=1}^{\infty} n^{-\frac{3}{2}} \frac{\exp[-\alpha^2/4n]}{1+(\alpha/\alpha_L)^2}. \tag{6}$$

The contribution of the ambiguity to the intensity is given by the integral of $\tilde{N}_{LP}(\alpha)$ over all frequencies; thus,

$$\frac{\overline{\phi^2_{LP}}}{(\Delta\omega)^2} = \sum_{n=1}^{\infty} \frac{n^{-\frac{3}{2}}}{4\pi^{\frac{1}{2}}} \int_{-\infty}^{\infty} \left\{\frac{\exp[-\alpha^2/4n]}{1+(\alpha/\alpha_L)^2}\right\} d\alpha, \tag{7}$$

where $\overline{\phi^2_{LP}}$ represents the mean-square low-passed ambiguity or phase fluctuations.

By taking Fourier transforms and using Parseval's relation, this can be integrated directly to yield

$$\frac{\overline{\phi^2_{LP}}}{(\Delta\omega)^2} = \frac{\pi^{\frac{1}{2}}}{4} \sum_{n=1}^{\infty} n^{-\frac{3}{2}} \alpha_L \left(1 - \operatorname{erf}\frac{\alpha_L}{2n}\right) \exp(\alpha_L^2/4n), \tag{8}$$

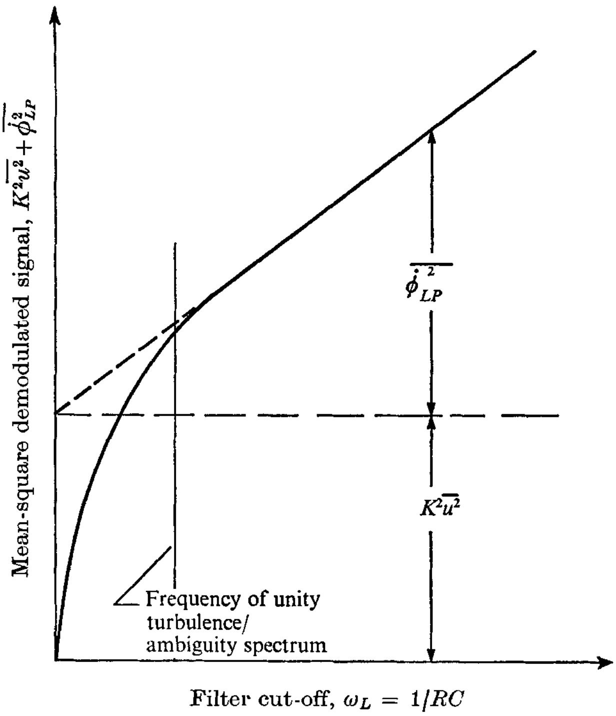

FIGURE 2. Plot of mean-square demodulated signal *vs.* cut-off frequency of low-pass filter. The extrapolation to zero cut-off (dashed line) corresponds to the true turbulence intensity. ($\alpha_L \ll 1$.)

where $\operatorname{erf}(x)$ is the classical error function (Abramowitz & Stegun 1964). If we assume that the filter cut-off frequency ω_L is small compared with the Doppler-ambiguity bandwidth $\Delta\omega$ (i.e. $\alpha_L \ll 1$), we can expand the right-hand side of (8) in powers of α_L and keep only the lowest-order terms. To third order, we have

$$\frac{\overline{\dot{\phi}^2_{LP}}}{(\Delta\omega)^2} \simeq \frac{\pi^{\frac{1}{2}}}{4}\sum_{n=1}^{\infty} n^{-\frac{3}{2}}\alpha_L\left[1-\frac{\alpha_L}{n\pi}+\frac{\alpha_L^2}{(2n)^2}\right]. \tag{9}$$

To first order, this reduces to

$$\overline{\dot{\phi}^2_{LP}}/(\Delta\omega)^2 \simeq \pi\tilde{N}(0)\,\alpha_L = 0{\cdot}368\pi\alpha_L, \tag{10}$$

where $\tilde{N}(0)$, the value of the ambiguity spectrum at the origin, is 0·368. This result is precisely that expected for a low-passed white-noise signal. By evaluating the sums in (9), it is easy to show that (10) is accurate to within 5 %, as long as $\alpha_L < \frac{1}{10}\pi^{\frac{1}{2}}$ (i.e. $\omega_L < \frac{1}{10}\pi^{\frac{1}{2}}\Delta\omega$). In a similar fashion (9) can be shown to be valid to within 2 % for $\alpha_L \leqslant 1$.

The form of (10) is particularly useful since the filter cut-off frequency appears linearly. To illustrate this, suppose that the filter cut-off frequency ω_L is chosen well below the Doppler-ambiguity bandwidth $\Delta\omega$ so that the linear approximation is valid. As we increase the cut-off frequency ω_L, through the frequency at which the spectrum of the ratio of turbulence to ambiguity is unity, the measured intensity will appear as in figure 2. The curve shown is similar to turbulence data

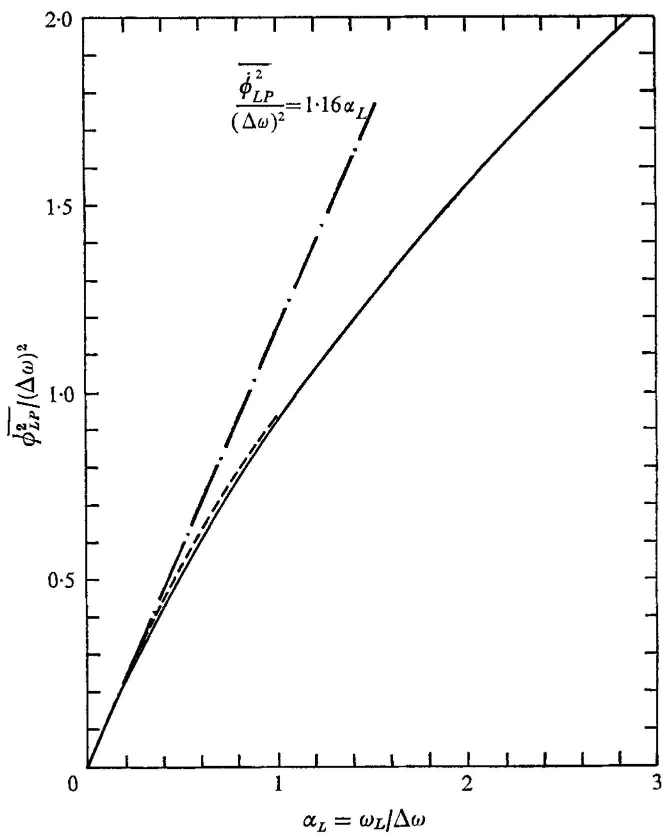

FIGURE 3. Mean-square filtered ambiguity *vs.* cut-off frequency of low-pass filter. ——, equation (8), exact; – – – –, equation (9), cubic; —·—, equation (10), linear.

acquired by Rowe (1973) using an LDV in rod bundles of a nuclear reactor. The linear region corresponds to only ambiguity being filtered, whereas the rapid roll-off for low cut-off frequencies indicates that the turbulence is also being filtered. It is clear from (9) that *the uncontaminated turbulence intensity is obtained by extrapolating the linear region to zero cut-off frequency,* the intercept being the true turbulence intensity. This result is not surprising since the ambiguity spectrum is effectively white for frequencies less than $\Delta\omega$; the application, however, has been previously overlooked. Note that the Doppler-ambiguity bandwidth $\Delta\omega$ does not need to be known *a priori*, and in fact can be determined from the slope of the linear portion of the curve.

The condition $\alpha_L < \frac{1}{10}\pi^{\frac{1}{2}}$ is not unduly restrictive since, by choosing the sampling volume small enough, we can make $\Delta\omega$ as large as we like (cf. George & Lumley, equation (4.2.8)). In some cases, however, this assumption will not be valid. In those cases, the extrapolation can still be performed by fitting (8) to the experimental data. In such cases, the Doppler-ambiguity bandwidth is required and must be estimated from the experimental data or calculated independently as in George & Lumley (1973) and Berman & Dunning (1973).

For convenience, dimensionless plots of (8)–(10) are shown in figure 3. The curve for (8) was obtained by summing over 1000 terms and adding an integral remainder.

It must be noted, in conclusion, that the filtering approach to LDV turbulence intensity measurements is not restricted to the use of a single-pole filter. In fact, the use of higher-order filters will relax somewhat the conditions on α_L. In addition, the technique is equally applicable to the measurement of non-axial intensities and Reynolds stress.

The turbulence intensity can always be obtained by measuring the power spectrum of the combined ambiguity and turbulence, fitting (2) for the ambiguity using the high frequency part of the measured spectrum (see figure 1*b*), subtracting the ambiguity from each measured spectral value and integrating the corrected spectrum. If the ambiguity spectrum is effectively white ($\Delta\omega$ large enough) and if the entire signal is low-pass filtered at a frequency well below $\Delta\omega$, the intensity correction can be made by subtracting from the measured intensity the product of a single measurement of the ambiguity spectral height and the equivalent noise bandwidth of the signal. This simplified procedure requires only a band-pass filter; however, it must be assured that the ambiguity spectrum is white, which generally requires several additional spectral measurements. Thus in most situations there is little effort, if any, eliminated by using this procedure in place of the moving filter described above.

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